كلية المشرق للعلوم والتكنولوجيا

امتحانات الفصل الدراسي (2) – 2008 /2009 م

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Q1

If $f(x)$ is defined for $x \geq 0$, then the Laplace transform of $f(x)$ is given by :

$$L\{f(x)\} = \int_0^{\infty} e^{-sx} f(x)\,dx$$

1) Prove that $L\{x^n\} = \frac{n}{s} L\{x^{n-1}\}$

2) Find :

$$i)\, L\{4x^2 - 5\sin 3t\} \qquad ii)\, L\left\{\left(1+e^{2x}\right)^2\right\} \qquad iii)\, L^{-1}\left\{\frac{(s+2)^2}{s^3}\right\}$$

3) Use the Laplace transform to solve : $y' - y = 1 \quad ; \quad y(0) = 0$

Q2

The unit step function is defined to be

$$u(t-a) = \begin{cases} 0 & ; \quad 0 \leq t < a \\ 1 & ; \quad t \geq a \end{cases}$$

1) Given :

$$f(t) = \begin{cases} 2 & ; \quad 0 \leq t < 3 \\ -2 & ; \quad t \geq 3 \end{cases}$$

i) Write $f(t)$ in terms of the unit step functions .
ii) By using (i) or any other method ,find the Laplace transform of $f(t)$

2) If $a > 0$ prove that $L\{f(t-a)u(t-a)\} = e^{-as} L\{f(t)\}$
3) Find $L\{e^{-2t}\cos 4t\}$

Q3

The convolution of $f(t)$ *and* $g(t)$ is defined as

$$f * g = \int_0^t f(\tau) g(t-\tau)\,d\tau$$

1) How we can find Laplace transform of the convolution of two functions ?
2) Is the convolution of two functions commutative ?
3) Evaluate $\int_0^t e^{\tau} \sin(t-\tau)\,d\tau$

Q4

Two functions f_1 *and* f_2 are said to be orthogonal on interval $[a,b]$ if the inner product of them equals zero .

1) Determine if the given functions are orthogonal on $[0,2]$ or not

$$f_1(x) = e^x \quad , \quad f_2(x) = xe^{-x} - e^{-x}$$

2) Show that the set $\{1, \cos x, \cos 2x, ...\}$ is orthogonal on the interval $[-\pi, \pi]$

3) Expand

$$f(x) = \begin{cases} 0 & ; \quad -\pi < t < 0 \\ 1 & ; \quad 0 \le t < \pi \end{cases}$$

in a Fourier series .